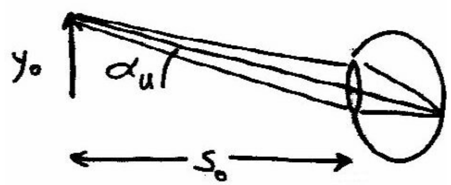
Magnifying Glass (simple magnifier)

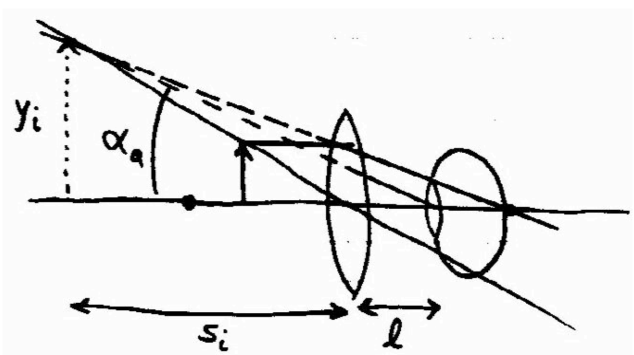
unaided eye: image formed on retina.



- get clear image only for $s_o > d_o$

i.e. object at near point ($d_o \approx 25$ cm)

positive lens: form virtual, erect image ($s_o < f$), with $|s_i| > d_o$:



- rays appear to come from virtual image, and are focussed by eye.

- compare angles subtended by "unaided" image, α_u, with that of "aided" image, α_a;

(paraxial) $\alpha_u \approx \dfrac{y_o}{d_o}$, $\alpha_a \approx \dfrac{y_i}{|s_i|+\ell}$

(max value, $s_0=d_0$)

magnifying power $MP \equiv \dfrac{\alpha_a}{\alpha_u} = \dfrac{y_i}{y_o}\dfrac{d_o}{|s_i|+\ell}$

but $\dfrac{y_i}{y_o} = -\dfrac{s_i}{s_o} \Rightarrow MP = -\dfrac{s_i}{s_o}\dfrac{d_o}{\ell - s_i}$

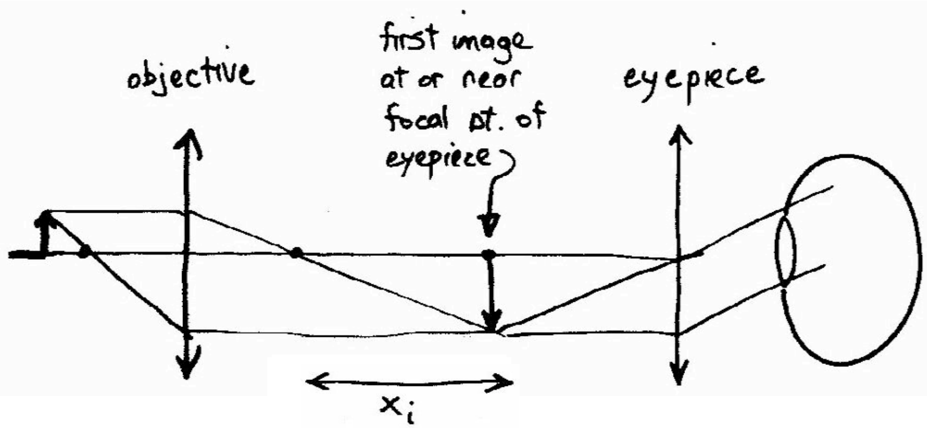
get various MP depending on s_i and ℓ; max MP for ℓ =0, and take $|s_i| \to \infty$ (for relaxed eye) yielding s_o=f $\Rightarrow$

$$MP \to \frac{d_o}{s_o} = \frac{d_o}{f}$$

eg. $f = 10$ cm, $MP = \dfrac{25\text{ cm}}{10\text{ cm}} = 2.5\times$

larger MP limited by aberrations.

Microscope - 2 lenses $f_o << f_e$



$M_{T_o} = -\frac{x_i}{f_o}$ $x_i \equiv$ tube length, usually standard value of 16 cm

$\Rightarrow$ total $M_T = M_{T_o} \cdot MP_e = -\frac{16}{f_o} \cdot \frac{25}{f_e}$

eyepiece: compound lens - designed for comfortable eye location. MP=10 typical

objective: eg. $f_o = 3.2 \Rightarrow 5\times$ mag.

fixed x_i : $s_o = \frac{f_o s_i}{s_i - f_o} = \frac{f_o(x_i + f_o)}{x_i} = f_o + \frac{f_o^2}{x_i}$

$\Rightarrow s_o \approx f_o$; higher mag $\Rightarrow$ smaller f_o and s_o

Telescopes - A) Astronomical

- focusses rays from object $s_o \to \infty$

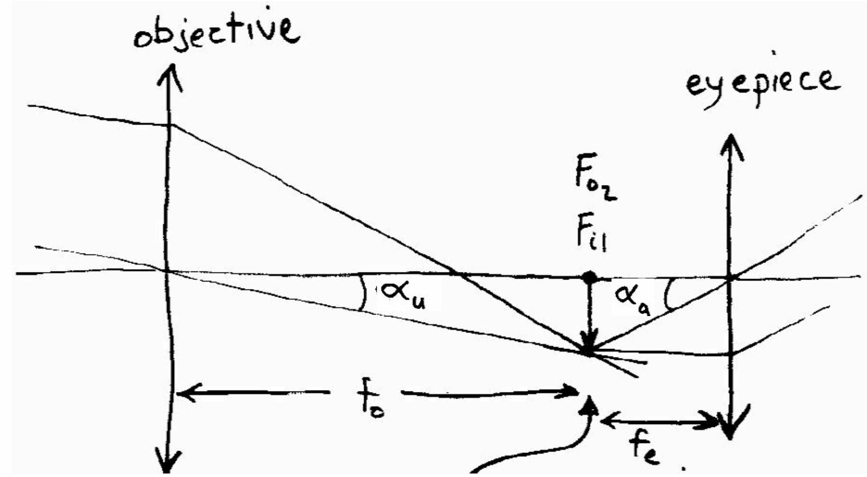


image from objective at F_{i_1} ; coincides nearly with F_{o_2}

inverted image

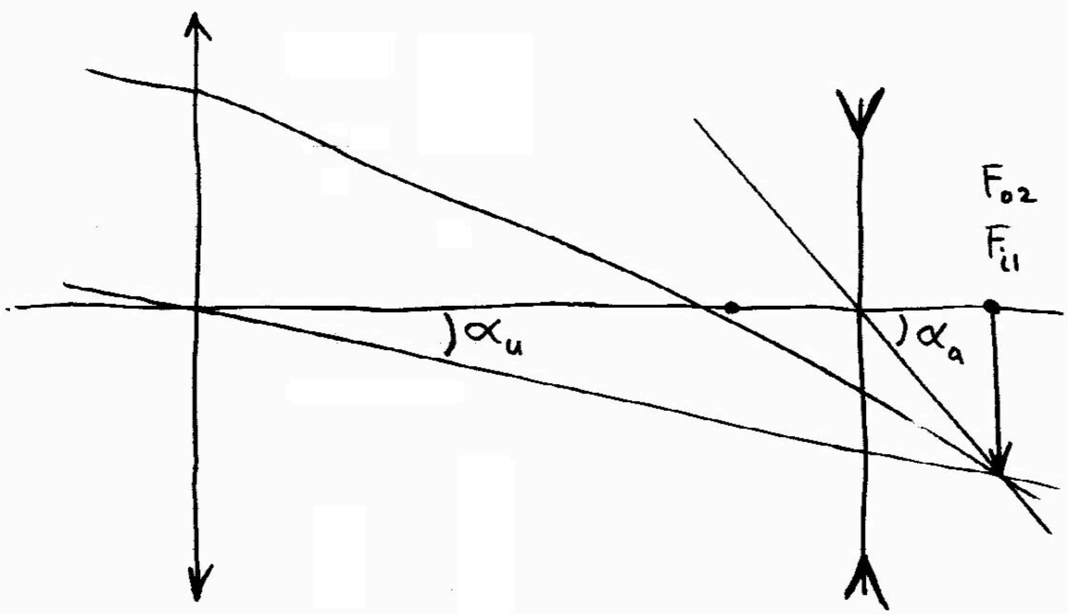
$$M \equiv \frac{\alpha_a}{\alpha_u} = -\frac{f_o}{f_e}$$

$f_o >> f_e$

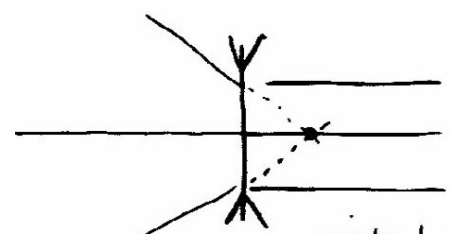
B) Galilean Telescope

$f_e < 0 \Rightarrow$ erect image

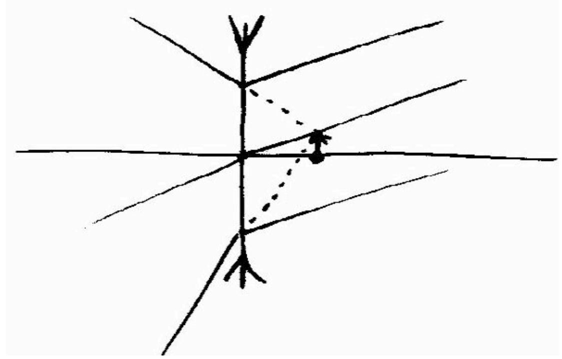
length $= f_o + f_e$, shorter than for astronomical telescope



recall: negative lens:



or



again, $M \equiv \dfrac{\alpha_a}{\alpha_u} = -\dfrac{f_o}{f_e}$

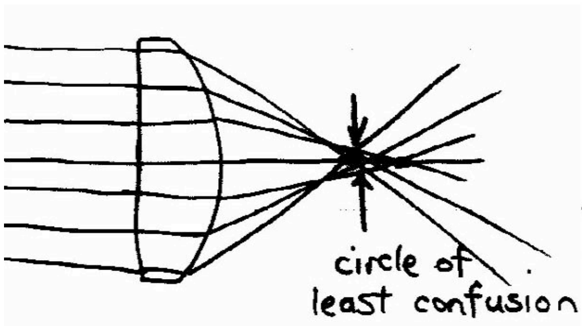
Aberrations: arise from –

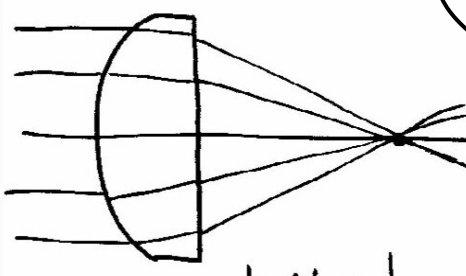
(a) paraxial approx ⟩ spherical aber., coma, astigmatism, curvature of field, . . .

(b) dep. of n on λ

examples:

- spherical aberration: nonparaxial rays are too strongly bent

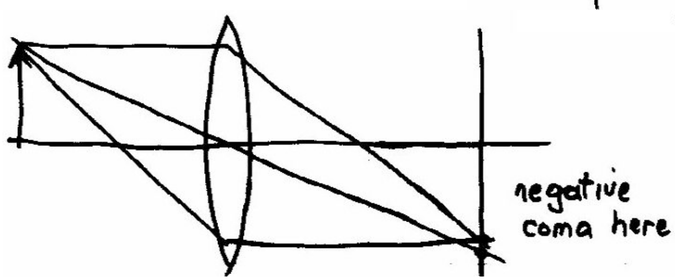




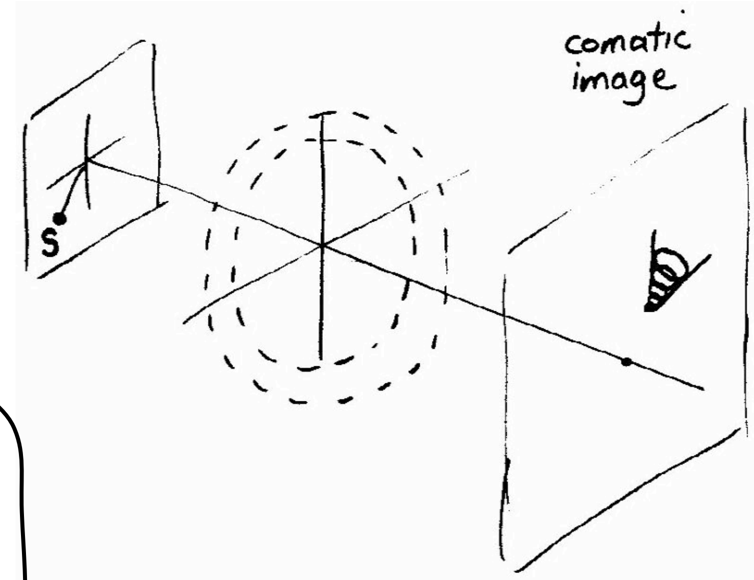
better!

- coma: arises when $y_0 \neq 0$

also depends on lens shape.

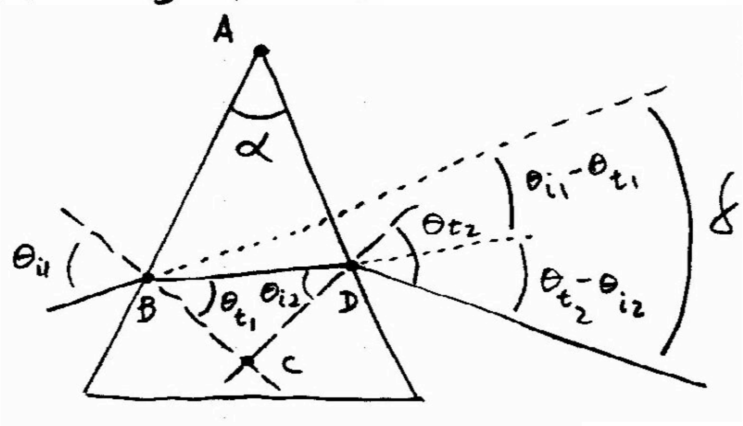


consider focussing of off-axis point:



Prisms - variety of types

dispersing prism:



given α, n, θ_{i1} determine θ_{t2}:

look at total deviation δ:

$$\delta = (\theta_{i1}-\theta_{t1}) + (\theta_{t2}-\theta_{i2})$$

- polygon ABCD has 2 right angles

$$\Rightarrow \alpha + \angle BCD = \pi$$

and $\angle BCD = \pi - (\theta_{t1}+\theta_{i2})$

$$\Rightarrow \alpha = \theta_{t1}+\theta_{i2}$$

thus $\delta = \theta_{i1}+\theta_{t2}-\alpha \quad (*)$

get θ_{t2} from Snell's law:

$$\theta_{t2} = \sin^{-1}(n\sin\theta_{i2}) = \sin^{-1}\Big(n\sin\underbrace{(\alpha-\theta_{t1})}_{\sin\alpha\,\underbrace{\cos\theta_{t1}}_{(1-\sin^2\theta_{t1})^{1/2}}-\cos\alpha\sin\theta_{t1}}\Big)$$

and $n\sin\theta_{t1} = \sin\theta_{i1}$

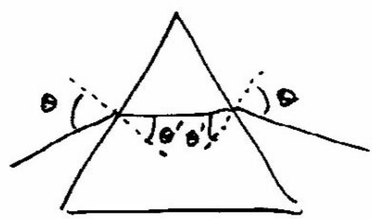
$$\Rightarrow \theta_{t2} = \sin^{-1}\left[\sin\alpha\left(n^2-\sin^2\theta_{i1}\right)^{1/2} - \cos\alpha\sin\theta_{i1}\right]$$

and get δ from $(*)$.

angle of minimum deviation:

symmetric ray:

$$\theta_{t1} = \theta_{i2} \equiv \theta'$$
$$\theta_{t2} = \theta_{i1} \equiv \theta$$



then $\delta = 2\theta - 2\theta'$

$\sin\theta = n\sin\theta'$

$\alpha = 2\theta' \quad \Rightarrow \theta' = \frac{\alpha}{2}$

$\theta = \frac{\delta+2\theta'}{2} = \frac{\delta+\alpha}{2}$

$$\Rightarrow \sin\left(\frac{\delta+\alpha}{2}\right) = n\sin\left(\frac{\alpha}{2}\right)$$

can solve for δ (given n) or n (given δ).